Zoe Holmes

How satisfactory is the de Broglie-Bohm theory (DBB) as a resolution of the measurement problem?

In this essay I will first set out the measurement problem as an explicit contradiction and show how the DBB denies premises necessary to set up the contradiction. I then consider and attempt to defend the DBB against suggested reasons for finding its resolution unsatisfactory. I end by concluding that while it is too early to whole heartedly assert the DBB there are reasons to be optimistic that it is on the right track.

What is the DBB theory?

In the DBB theory there are point particles (corpuscles) with definite locations at all times however their exact location is unknown and hence is instead described by a probability distribution. The wavefunction has a dual role as it provides the probability distribution of the positions of the particles at any time and plays a dynamic role as a real wave in 3n configuration space driving the motion of the corpuscles. The ingredients of the theory are as follows:

Ontology:

- Point Particles ("Corpuscles") with defined positions: q1, q2... qn
- The Quantum Wavefunction ("the pilot wave"): $\Psi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t)$ in a 3n-dimensional configuration space (n is the total number of particles in the universe).

Dynamics:

- The Wavefunction evolves unitarily via the Schrodinger Equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \sum_{i=1}^{N} -\frac{\hbar^2}{2m_i}\nabla_i^2\Psi + V\Psi$$

- The corpuscles trajectory is given by the guidance equation:

$$m_i\frac{d\mathbf{x}_i}{dt} = \nabla_i S \quad \text{where} \quad S = \hbar \operatorname{Im} \ln \Psi$$

Probabilistic Hypothesis:

- The probability distribution of the particles at an arbitrary time t is given by:

$$P(\mathbf{x}, t) = |\Psi(\mathbf{x}, t)|^2.$$

The DBB reproduces quantum mechanical predictions as follows. The probability rule gives DBB the same probability distribution for position as QM for any initial time t0. The guidance equation tells us how the two sides of the probability rule evolve and, by construction[1], ensures that the probability distribution for x at a later time t agrees with that of QM when evaluated using the probability rule. DBB gets the probability of the transitions during measurement in agreement with the collapse postulate of QM because of effective collapse. DBB gives the correct predictions for the probability of measurements of other qualities because all measurements can be considered to be ultimately measurements of position. The reason for this is that the outcome of any measurement (say, momentum, charge or spin) is ultimately decided by the time-dependent positions of a macroscopic number of atoms or electrons, which belong to a pointer or electrical circuit in the experimental apparatus.

[1] To define the trajectory of the particles we need a velocity vector. This can be obtained by taking the ratio of the probability current to the probability density. Probability current satisfies the QM continuity equation. $\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0$ gives $\mathbf{j} = \frac{i\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) = \frac{\hbar}{m}\operatorname{Im}(\Psi^*\nabla\Psi)$. $\dot{x}(t) = \frac{j(x,t)}{|\Psi(x,t)|^2}$. This simplifies to the guidance equation given above. Thus, given the guidance equation and a probability distribution at some time, we have the trajectory for the corpuscles at all future time. Therefore, given an initial probability distribution for the particles we can evolve the distribution forward in time and use the probability rule to give us the probability that the trajectory of a particle has lead it to be in a particular location at a later time t.

this is important - don't relegate it to a footnote

be clear as to whether this rule is imposed at all times, or only at one time

What is the measurement problem?

(When I refer to quantum mechanics, QM, I am referring to the rough more or less un-interpreted formulation taught in university courses and used experimentally to make incredibly accurate predictions)

Careful with this – as we've discussed, it's not clear what this theory is

The measurement problem results from a contradiction between the linearity of the dynamics of Quantum Mechanics and its measurement kinematics when combined with a minimal conception of a good quantum measuring device and, most importantly here, the assumption that the wavefunction for a system provides a complete description of the system.

Quantum Mechanical Postulates:

A) Measurement Kinematic Postulate: If Q is an observable the *post measurement state* the wavefunction of the system will be the *eigenstate* corresponding to the eigenvalue measured.

B) Dynamical Postulate: Time evolution of the wavefunction in QM is a *linear* map from state to state.

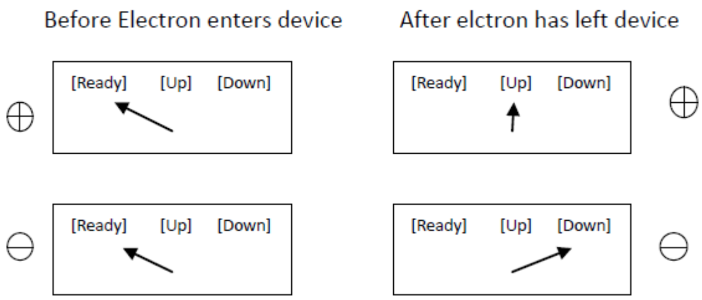
cf take discussion in week 1 – this rule actually plays v. little part in applications of QM

Additional Implicit prerequisites:

Since with these prerequisites the theory is inconsistent, it's not clear in what sense they can be implicit!

1) The wavefunction is a complete description of the state of a system.
2) Basic Conception of a measuring device: a good measuring device is accurate.
3) Quantum Mechanics is a universal and fundamental theory
4) Weak Physicalist Postulate: The description of the behaviour of large objects must be consistent with the laws governing the behaviour of the smaller objects of which they consist.

A good measuring device must be accurate. If we feed in a system in a defined state the measuring device would indicate that the system is in that state. Furthermore, given QM is a fundamental theory we should be able to assign pointer states to the measuring device. As such, we can formalise a simple measurement procedure as follows:



$$|+\rangle \otimes |ready\rangle \rightarrow |+\rangle \otimes |up\rangle$$

$$|-\rangle \otimes |ready\rangle \rightarrow |-\rangle \otimes |down\rangle$$

The seeming contradiction is generated when we consider what happens when you feed in a superposition to the measuring device. From, the linearity of the dynamics, and our conception of a good measuring device:

$$(\alpha|+\rangle + \beta|-\rangle) \otimes |ready\rangle \rightarrow \alpha|+\rangle \otimes |up\rangle + \beta|-\rangle \otimes |down\rangle$$

According to dynamics is no way to get the system into an eigenstate of an observable if it is not already in one. This contradicts A. The measurement kinematics state that post measurement of the system will be in an eigenstate of the observable being measured.

QM draws a line between the normal dynamics of QM and the measurement kinematics during which the wavefunction can be said to collapse onto a single eigenstate. This works in practise but as the concept of measurement is vague and not mathematically defined this means that QM is based on untenably vague foundations.

in what sense are they untenable?

How does DBB avoid the measurement problem?

In general terms the DBB avoids the measurement problem because implicit in the above construction of the problem is the assumption that the wavefunction is a complete description of the post measurement system. DBB denies this. According to DBB a complete description of the after measurement system includes, in addition to the wave function, the value of the variable that registers the result.

In terms of the wavefunction, DBB accepts B (that the *wavefunction* evolves linearly) but denies A (that the post measurement wavefunction of the system will strictly be the *eigenstate* corresponding to the eigenvalue measured). In this way, the DBBs use of the wavefunction does not lead to the contradiction set out above.

"Contradiction" seems too strong: rather, it's a (perhaps unacceptable) ambiguity in the collapse law

DBB accounts for the fact we do not observe macroscopic superpositions and theory reproduces QM predictions once the corpuscles are taken into account. All particles, those of the corpuscles of the system and those of the measuring device alike, although we do not know these positions, have defined positions and follow deterministic trajectories as given by the guidance equation. This explains why we obtain a macroscopic measurement of one value and not a superposition.

The DBB still has to account for the fact that we treat the wavefunction as collapsing post measurement if it is to agree with QM predictions. It does this by appeal to the way that the wavefunction of the subsystem post measurement effectively collapses. This allows us to treat other branches of the universes wavefunction as sufficiently far away in phase space that we do not have to worry about the outgoing branches of the way function re-overlapping and interfering in the future. As such, we can effectively ignore the other branches when considering the wavefunction of the subsystem post measurement. Given this, for all practical purposes, the results of measurement are irreversible and so we say that the wavefunction of the subsystem has effectively collapsed onto the branch of the original wavefunction associated with the eigenvalue of the observable measured. ok

How does the explanation go? – what is the relation between corpuscles & observations?

Effective collapse is entailed by decoherence. Decoherence is the process by which vast numbers of macroscopic degrees of freedom get entangled with the configuration space of the environment. You can derive the effective collapse of a subsystem wavefunction mathematically. It is a physical process which takes a finite amount of time (the more macroscopic the object, the quicker it gets entangled with the environment, the quicker the branching process). ok

Why might this resolution, provided by DBB, not be satisfactory?

Yeah, on reflection it wasn't the best choice of question, sorry...

It is tricky and vague, to say the least, to define degrees of satisfaction. It is easy to make these sorts of judgements when dealing when they are comparative. My general approach will be to consider ways in which DBB might be unsatisfactory, argue that none of these factors make DBB definitively unsatisfactory, before comparing the theory to the closest alternatives.

A discussion of the correct philosophy of science is beyond the scope of the essay and so to make my position clear: I advocate generally a cautious realism. Most theories (but not all) purport to genuinely refer and by and large, the well confirmed parts of well confirmed theories of mature sciences (not all of all our posited theories) are approximately true. fair enough

Petty Objections:

- The DBB has strayed quite a way from the most intuitive hidden variable theory that one might have thought one would like to replace QM. The wavefunction is not a probability distribution assigned to a bunch of underlying variables including all the different classical properties assigned to a system. The only definite values that particles have under DBB are positions.

NB this is linked into the question of the theory/world relation: if Ψ is unphysical then of course it does not represent anything

in general this is not appropriate language for written philosophy

- The action reaction principle is violated because the wavefunction acts of the corpuscles but the corpuscle does not act back on the wavefunction.
- The wavefunction is a wave in a 3n dimensional configuration space. It is bizarre defining such an entity a dynamic role.

My response to all of the above is: “who cares?”. They are pointing out unusual features of the theory which we did not have in classical theories but are inessential.

Objection 1: What’s the point in the Corpuscle?
It might be argued that DBB’s corpuscles lack a clear role and merely act as pointers which determine which sections of the wave function to keep and which sections of the wave function to chuck out on measurement. On this reading it is unclear what the particles actually do. They sit inside a real wave function and only come into play when something is measured. They then indicate which part of the wave function carries on to be relevant to the real world and the rest of the wave function can just carry on evolving but containing no particles. ✓

but you haven't given the explanation

Response:
DBB is a well-defined theory and solves the measurement problem by introducing corpuscles. The corpuscle has explanatory power; it explains why only one branch of the wavefunction ends up occupied. Both aspects of DBBs ontology are necessary to replicate the well confirmed predictions of QM while solving the measurement problem.
Furthermore, this dual ontology has further explanatory power. It can explain wave particle duality. If you take the two slit experiment the corpuscles account for the definite location of the particles identified by the fact that each particle must go through one of the two slits because the interference pattern does not appear if only one slit is open. The existence of a real wavefunction, which plays a dynamic role, accounts for the interference patterns that we observe and from which we infer entities also have a wavelike nature.

(i) does this say anything beyond "it solves the MP"? (ii) does it rely on the fact that the 1-particle wf lives in 3D space?

Objection 2: But what are the non occupied branches of the wavefunction?
This follows on from the previous objection. If you grant that the corpuscle denotes the real branch post measurement it still remains to be explained the ontological status of the other remaining branches of the wavefunction in configuration space. This leads to the Everett in denial criticism. It can be argued that the only difference between the Everett interpretation and DBB is that DBB adds the corpuscles. Given that the function of the wavefunction in the two theories is the same, and given functionalism (that the ontology of an entity is determined by its function), and given an Everetian conception of the ontology of the unactualised branches of the wavefunction, we are lead to the conclusion that DBB theorists are Everettian’s in denial and the branches of the wavefunction not occupied by the corpuscle represent a multiplicity of physically real worlds.

Response: The above objection does not make much sense when considered on the DBB ~~theories~~ theory's own terms. Only if you accept the interpretation of branches of the wavefunction as physically real worlds can you accuse DBB of wrongly denying the existence of these worlds.
The DBB theorist can insist that the wavefunction is real and plays a dynamic role but is just not the sort of thing that can make up macroscopic objects. Any wave of a vibrating string can be written as a sum of its oscillating modes. However, in this case the true ontology consists of the total displacement of the string as a function of position and time. It would not be asserted that there exists a multiplicity of physical strings each vibrating in a single mode. In the case of the vibrating sting the eigenfunctions and eigenvalues have mathematical meaning only. This analogy is imperfect as the branches/eigenvectors of the wavefunction play more of a dynamic role than the modes of a string as the overlapping of branches overlap accounts for interference. It would seem that the ontology of the wavefunction, in particular the unoccupied branches, is something different to any classical analogue. It is desirable that more be said of the ontology of the wavefunction but its oddity is no reason to reject it as the above

indeed

As a rule this means something's gone wrong with your structure how is this compatible with

comparison to the description of a vibrating string in terms of normal modes suggests we need not conclude that there are a multitude of physical worlds.

Objection 3: The theory is underdetermined: Firstly, you cannot directly observe the trajectories directly because you cannot determine the location of the corpuscles any more precisely than the wave packet they lie in. Secondly, any other hidden variable theory with an alternative dynamic equation which produces the same macroscopic result would be empirically indistinguishable from DBB. For example, the guidance equation is defined a probability current divided by probability density; however, the exact form of probability current is underdetermined. J is the solution to the continuity equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$ from which we generally conclude that $\mathbf{j} = \frac{i\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) = \frac{\hbar}{m}\mathrm{Im}(\Psi^*\nabla\Psi)$. However, any J' which equals J plus the curl of a time independent vector function would also be a solution to the continuity equation (because the divergence of the curl of a function is always zero). Thus, as J appears in the guidance equation, there are numerous empirically equivalent but inconsistent guidance equations.

Response: In response to the first point it is true that we cannot directly observe trajectories but that does not mean that they do not exist. There is a good reason why we cannot directly observe the trajectories: any interaction involved with detecting the trajectories would influence the particles future motion. Trajectories can play an explanatory role both in explaining the results of any measurement of a QM system and resolving the measurement problem. Thus, if you accept that explanatory power is truth conductive, there is evidential support for their existence.
In response to the second point, the under determination with respect to the exact form of the hidden variable dynamic equations can potentially be resolved by appealing to extra theoretical virtues. Certainly this is the case in regards to the under determination of the form for J. It can be shown that the usual equation for J (stated in this essay) has the simplest mathematical dependence on the wavefunction and possesses Galilean symmetry.
The importance that is lent to the criticism of under determination will depend on your philosophy of science. However, if philosophy of Quantum Mechanics is to be possible at all, given that we are largely considering empirically equivalent theories, one must accept that extra empirical values play a role in theory choice and as such any argument from under determination is weakened.

cf tutorial discussion

Objection 4: DBB is non local.
The DBB is non local in the sense that the wavefunction explicitly depends on the location of all corpuscles in the universe. As the guidance equation incorporates the wavefunction the trajectory of any corpuscle will depend on the positions of other corpuscles with which its wavefunction is non-separable.

also in the sense that there is direct action at a distance

Response: Bell's inequality shows that any theory which agrees with the prediction of quantum mechanics (and accepts that only one result obtains per measurement) will have non-local features. Non locality is undesirable as the existence of faster than light signals violates Lorentz covariance. Incompatibility with other fundamental theories is certainly an undesirable property of a theory. Nonetheless, given that QM is incompatible with the current best theory of macroscopic objects, General Relativity, this does not seem a knockdown argument. Particularly, given that the no signalling theorem ensures that it is not possible to send macroscopic signals and so there is not the potential for particularly bizarre, but arguably not logically impossible, things like macroscopic closed causal loops.

There wouldn't be anyway – DBB presupposes a preferred reference frame

This is overstated – we don't currently have a quantum theory of gravity but that doesn't mean QM and GR are incompatible

Zoe Holmes

How satisfactory is the DBB solution to the measurement problem?

The DBB solves the measurement problem and I have found no reason to accept that it is untenable. Nonetheless it has some unusual and potentially undesirable traits; in particular a dual ontology with bare point particles and a wavefunction with an odd ontology and the theory is fundamentally non-local. How satisfactory is this theory? It is easier to answer this comparatively.

As DBB solves the measurement problem it is certainly more satisfactory than any theory which does not. Such theories include as the non-interpretation that we are taught in the second year or the "Copenhagen interpretation" (whatever that actually is) both of which are fundamentally ill-defined because they depend on a clear dividing line between the system and macroscopic measuring device but such distinctions defy sharp and precise formulation. It is beyond the scope of this essay (given that I haven't done any of the reading on it yet so don't have a clue) how the DBB weighs up against the Everett Interpretation and so I will leave this an open question. However, it should be noted, that any theory, such as the Everettian Interpretation which is not prescribed non-local features by the Bell non-locality theorem has some advantage over DBB. fair enough

It is worth noting, given that I have concluded that DBB is superior to the commonly used version of QM, that there are historical reasons which could explain why this version of QM was adopted over DBB. In the 1920s to 1950s when the foundations of QM were being established, logical positivism was the prevailing philosophy of science. This advocated understanding the world entirely in terms of directly observable quantities and so disapproved of hypothesising unobserved "hidden variables". Conditional claims are problematic but, given the lack of definitive arguments against DBB and given it solves the measurement problem, it is not entirely unconceivable that had the foundations of QM been developed in a different climate, an approach closer to DBB might have been adopted. With the exception of non-locality, the main argument against DBB stem to difficulties accepting oddities of its dual ontology. Had a version of DBB been adopted and developed, what we take to be oddities in the theory might be more familiar and hence easier to accept. cf Jim Cushing; both advocating (something like) this position

I advocate a cautious realism in which it is not necessary to accept that all our theories are approximately true. If the DBB were the only consistent account of QM would it win out against either an operationalist account of QM or remaining temporarily agnostic with regard to its truth value? The DBB arguably wins out against an operationalist account as it is unclear whether an operationalist account on the wavefunction is either coherent or warranted within a general realist framework. Given that QM is currently incomplete as it is incompatible with General Relativity a degree of agnosticism is required to any interpretation until further attempts have been made to extent the theory. DBB in particular has only been formulated in a non-relativistic context to and so this is an additional extension which is required before the theory can be fully endored. Although some people suspect that pilot-wave concepts cannot be sustained in realm of quantum field theories this seems premature given that QM is also incomplete. Thus, although it would be premature to endorse DBB entirely, mitigated optimism that it's on the right track does is not entirely unreasonable.

I don't understand this claim

The deBroglie Bohm Theory resolves the measurement problem by denying that Quantum mechanics is complete. Its appeal to corpuscles accounts for which branch of the wavefunction is actualised during measurement. Its insistence that the wavefunction is a real dynamics entity accounts for all interference effects. The theories biggest deficit is its non-locality but in that respect it stands no worse than any other interpretation of QM with the exception of the Everett interpretation. There remains more to be said on the ontology of the wavefunction and the theory needs extending into relativistic field theory. If this can be done, and if the Everett interpretation does no better, then DBB is a satisfactory solution to the measurement problem.

In general this is a very good essay - you cover a lot of ground and make a clear philosophical case, even though your argument over-reaches in places.

The main philosophical lacuna is a discussion of just how the dBB corpuscles are related to observed reality - you assume uncritically that (e.g.) observations, or macro-objects, supervene on corpuscle positions, yet this is exactly what is denied by, e.g., Harvey's 1996 paper or his and my joint paper. (You touch on this criticism but don't give the positive case for regarding the corpuscles thus.